An Accurate Magnetic Equivalent Circuit Model for Analysis of Surface Mounted Permanent Magnet Motors

Jawad Faiz, *Senior Member, IEEE*, and Farhad Rezaee-Alam

Centre of Excellence on Applied Electromagnetic Systems, School of Electrical and Computer Engineering,
College of Engineering, University of Tehran, Tehran 1439957131, Iran

This paper presents an accurate and at the same time fast technique for performance analysis of surface mounted permanent magnet motors based on magnetic equivalent circuit (MEC) model. The main drawback of the MEC is its poor modeling of air-gap. Therefore, in order to provide accurate permeances of the air-gap for inclusion in the MEC model, the conformal mapping (CM) method is used. This combined model can precisely consider the magnetic saturation and the harmonics of air-gap permeances. To consider the impact of demagnetization of permanent magnets (PMs) and core losses, PMs and core are divided into many elements, and the magnetic scalar potential is calculated for all vertices. The accuracy of this technique is proven by comparing the results obtained through this combined model with corresponding results obtained from the conventional MEC, and finite elements method.

***Index Terms*—Air-gap Permeance, conformal mapping, magnetic equivalent circuit, permanent magnet.**

I. Introduction

THE ACCURATE modeling of electrical machines is possible by using the finite element method (FEM). Nowadays, the software packages facilitate the use of FEM. However, the software interfaces make the engineers lacking the knowledge of physics of the related problem. The long simulation time is another drawback of FEM [1]. For this reason, the analytical and semi-analytical methods such as, magnetic equivalent circuit (MEC) [2], conformal mapping (CM) [3], winding function theory [4], subdomain model [5], and field reconstruction method [6] have been introduced for magnetic field computation. However, these analytical techniques usually have imperfections in modeling the air-gap, magnetic saturation, or both. To enhance the accuracy of analytical methods and extend their applications, these analytical models may be combined to form the hybrid analytical models (HAM) [7]. In this paper, a new HAM based on MEC and CM is presented.

II. Air-gap Permeances

In conventional MEC, the air-gap permeances have been approximated using some equations with a low accuracy [2]. Tooth contour method (TCM) was also used to this purpose [8]. However, TCM needs cumbersome manual calculations. Since the slotting effect on the flux tubes in the air-gap can be accurately modeled using the conformal mapping (CM) method, the air-gap permeances are first calculated by CM. Fig. 1 shows a zoomed view of an exact MEC model of an analyzed SMPM motor. For simplicity, a few number of the air-gap permeances are shown in Fig. 1.

The air-gap permeances can be divided into two following general groups:

a) Mutual air-gap permeances:
   i. Permeances of PM-tooth (green in Fig. 1).
   ii. Permeances of PM-slot opening (red in Fig. 1).
   iii. Permeances of tooth-interpole (blue in Fig. 1).

b) Leakage air-gap permeances:

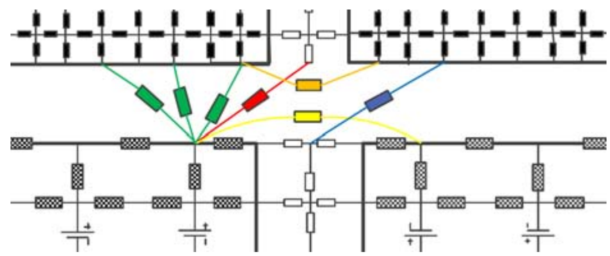

Fig. 1. An exact MEC model of SMPM motor.

   i. Permeances of tooth-tooth (orange in Fig. 1).
   ii. Permeances of PM-PM (yellow in Fig. 1).

In fact, the magnetic coupling between the rotor and the stator can be modeled by the mutual air-gap permeances.

A. Calculation of Mutual Air-gap Permeances

The following steps should be performed for calculating the mutual permeances in the air-gap:

✓ A virtual turn-function is considered for each element of the stator teeth, slot openings, PMs, and inter-ploes in the surface of the air-gap. These virtual turn-functions have the unit amplitude in an angular range equal to the width of respective element $\left(n_t(\varphi), n_{it}(\varphi), n_p(\varphi), n_{ip}(\varphi)\right)$.

✓ To consider the slotting effect in the calculation of the air-gap permeances, the radial component of specific complex permeance should be calculated in the middle of the air-gap $\left(\lambda_r(\varphi)\right)$. Fig. 2 shows the components of the specific complex permeance due to the stator slotting.

✓ The base value for each component of mutual air-gap permeances is calculated in slotless condition (G_b).

✓ The mutual air-gap permeances between the stator and rotor elements in the surface of the air-gap are then calculated as

$$G_{x,y} = G_b . \int_0^{2\pi} n_x(\varphi) . n_y(\varphi) . \lambda_r(\varphi) d\varphi \tag{1}$$

where $G_{x,y}$ is the mutual air-gap permeance between x^{th} element of stator and y^{th} element of rotor. Fig. 3 shows the mutual permeances between the elements of stator and rotor in the surface of the air-gap.

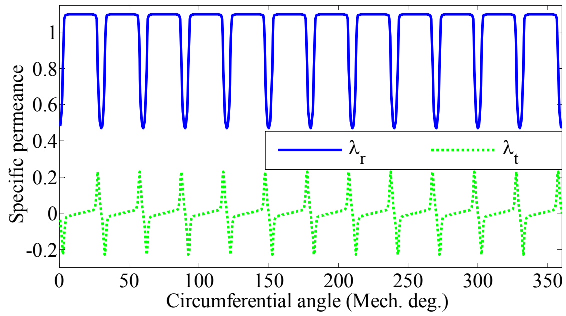

Fig. 2. Specific permeance due to the slotting effect.

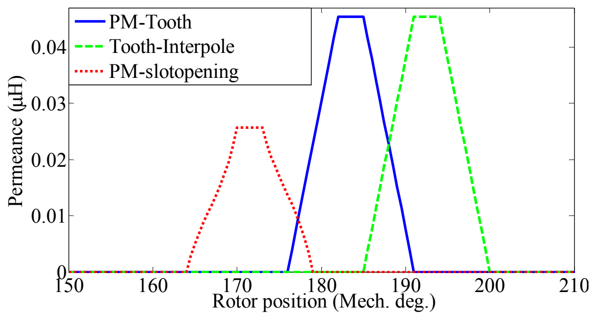

Fig. 3. Mutual air-gap permeances.

B. Calculation of Leakage Air-gap Permeances

To calculate the air-gap leakage permeance between the two adjacent PMs or teeth, the tangential components of the specific air-gap permeance due to the PM saliencies and the stator teeth should be calculated, respectively $(\lambda_{t,pm}, \lambda_{t,teeth})$.

$$G_{l,pm} = \frac{G_{b,p}}{2} \cdot \int_0^{\tau_p} \frac{|\lambda_{t,pm}|}{\lambda_r(\varphi)} d\varphi \quad (2)$$

$$G_{l,tooth} = \frac{G_{b,t}}{2} \cdot \int_0^{\tau_s} |\lambda_{t,teeth}| d\varphi \quad (3)$$

where $G_{l,pm}$ and $G_{l,tooth}$ are respectively the leakage permeances between the two adjacent PMs and two adjacent teeth. $G_{b,p}$ and $G_{b,t}$ are the base permeances calculated in slotless condition. τ_p and τ_s are the pole pitch and the slot pitch, respectively.

Fig. 4 shows the leakage permeance between the two adjacent PMs with the rotation of rotor in one slot pitch. For surface mounted permanent magnet (SMPM) motor, $G_{l,tooth}$ is constant. However, $G_{l,pm}$ varies with the rotation of rotor due to the slotting effect. After the calculation of air-gap permeances for each rotor position, the permeance and the magneto motive force sources (if any) are calculated for all elements of the core and PMs, while the core saturation effect has been considered. The nonlinear system of

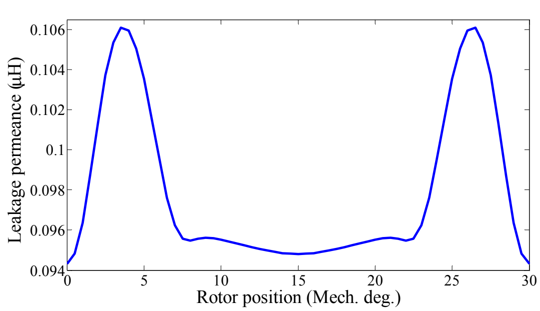

Fig. 4. Air-gap leakage permeances between two adjacent PMs.

algebraic equations for an SMPM motor including the node potential and electrical equations are then created and solved. Fig. 5 compares the total flux-linkage of phase A obtained through the conventional MEC, HAM, and FEM. Table I shows the some parameters of typical SMPM motor.

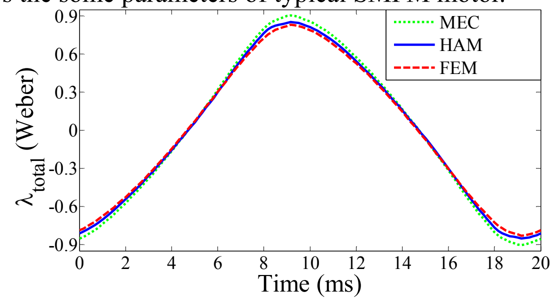

Fig.5. Total flux-linkage of phase A.

TABLE I
MAIN PARAMETERS OF SMPM MOTOR

Parameter	Value	Parameter	Value (mm)
Number of pole pairs, p	2	Stator outer diameter	130
Number of slots, Q_s	12	Stator inner diameter	75
Magnet remanence, B_r(T)	0.96	Active length, L	65
Relative recoil permeability, μ_r	1.07	Air gap length, g	1
Pole arc coefficient, αp	0.9	Magnet thickness	3.5

III. Conclusion

This paper presents an accurate MEC which precisely models different parts of electric machines including air-gap, PMs, and the core. The air-gap permeances used in precise MEC model are calculated by the CM method in short time, without any knowledge about the air-gap flux distribution, but taking into account the slotting effect. Therefore, the proposed MEC model can accurately predict the magnetic behavior of electric machines.

IV. Acknowledgement

The authors would like to thank Iran's National Elites Foundation (INEF) for financial support of the project.

References

[1] M. Aydin, and M. Gulec, "A new coreless axial flux interior permanent magnet synchronous motor with sinusoidal rotor segments" *IEEE Trans. Magn.*, vol.52, no.7, **Article #:** 810520, Jul. 2016.

[2] J. Faiz, and M. Ghasemi-Bijan, "Estimation of induction machine inductances using three-dimensional magnetic equivalent circuit" *IET Electr. Power Appl.*, vol.9, no.2, **pp. 117-127**, Mar. 2015.

[3] K. Abbaszadeh, and F. Rezaee Alam, "On-load field component separation in surface-mounted permanent-magnet motors using an improved conformal mapping method" *IEEE Trans. Magn.*, vol.52, no.2, **Article #:** 5200112, Oct. 2015.

[4] J. Faiz, and I. Tabatabaei, "Extension of winding function theory for nonuniform air gap in electric machinery" *IEEE Trans. Magn.*, vol.38, no.6, **pp.** 3654 - 3657, Jan. 2003.

[5] T. Lubin, S. Mezani, and A. Rezzoug, "2-D Exact analytical model for surface-mounted permanent-magnet motors with semi-closed slots" *IEEE Trans. Magn.*, vol.47, no.2, **pp.** 479 - 492, Feb. 2011.

[6] C. Lin, W. Wang, M. McDonough, and B. Fahimi, "An extended field reconstruction method for modeling of switched reluctance machines" *IEEE Trans. Magn.*, vol.48, no.2, **pp.** 1051 - 1054, Jan. 2012.

[7] A. Hemeida, and P. Sergeant, "Analytical modeling of surface PMSM using a combined solution of maxwell's equations and magnetic equivalent circuit" *IEEE Trans. Magn.*, vol.50, no.12, **Article #:** 7027913, Jun. 2014.

[8] V. A. Kuznetsov, "Numerical modeling of electromagnetic process in electromechanical systems", *COMPEL-Int. J. Comput. Math. Elect. Electron. Eng.*, vol. 22, no. 4, **pp.** 1142-1154, 2003